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1984 J. Phys. A: Math. Gen. 17 L859

(http://iopscience.iop.org/0305-4470/17/15/008)

J. Phys. A: Math. Gen. 17 (1984) L859–L861. Printed in Great Britain

LETTER TO THE EDITOR

Speculations on self-avoiding surfaces in fractals—a mean field treatment

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Received 7 August 1984

Abstract. We estimate the exponents characterising the self-avoiding surfaces using an approximation in the framework of a Flory-type theory. We find for planar self-avoiding surfaces embedded randomly in a fractal of dimensionality D': $\nu = 3/(4+D')$; for random surfaces of fractal dimension D embedded in a Euclidean space of dimensionality d: $\nu = 3/(2D+d-2)$; and for fractal surfaces embedded in a structure of fractal dimensionality D′: $\nu = 3/(2\mathrm{D}+\mathrm{D}'-2)$.

Recently, Maritan and Stella (1984) have attempted to generalise the excluded volume phenomena of random walks to that of planar random surfaces (plaquettes) in euclidian space. They have argued that the Flory theory compares very well with the real space renormalisation group for the exponent for the radius of gyration of the self-avoiding surfaces (SAS). Here, we extend it further (i) for the planar random surfaces embedded in fractal of dimensionality D', (ii) for the fractal (of dimensionality D) random surfaces embedded in euclidian space of dimensionality d and (iii) for the fractal (of dimensionality D) random surfaces embedded in a fractal of dimensionality D'.

(i) *Planar random surfaces embedded in the fractal.* A plaquette is a small basic unit to build up a large self-avoiding surface object. We restrict it to infinite fractals of dimensionality D' (e.g. an incipient infinite percolating cluster (IIC)) embedded in an euclidian hypercubic lattice of dimensionality $d \geq 3$. These plaquettes observe certain rules to explore different conformations on the fractal. Two adjacent plaquettes may join each other either at a common point or along a common side (i.e. a line), such that the surfaces (i.e. planes) explore randomly the available space. The excluded volume restriction means that two plaquettes cannot occupy the same area.

Let R be the linear dimension of such a SAS object. We follow Maritan and Stella (1984) in assuming that the surface area A here scales like the chain length in a self-avoiding walk (SAW) chain for calculating the elastic free energy i.e.

$$F_{\mathrm{el}} \propto A^2/A_0^2 = R^4/N \tag{1}$$

where N is the number of plaquettes, $A_0 \propto R_0^2 \propto \sqrt{N}$ is the ideal (gaussian) area. The repulsive free energy due to the exclusion of plaquettes is given by

$$F_{\mathrm{rep}} \propto N^2/R^{D'} \tag{2}$$

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in our Flory-type mean field theory. Minimising the total free energy $F = F_{el} + F_{rep}$ with respect to R we obtain

$$R \propto N^{\nu} \tag{3}$$

$$\nu = 3/(4+D'). \tag{4}$$

An alternative to equation (1) is to replace the denominator N by N^{2k} with $k = D_s/2D'$ since a random walk on a fractal moves a distance varying as t^k in time t (Rammal and Toulouse 1983, Gefen *et al* 1983, Pandey *et al* 1984); D_s is the spectral or fracton dimension. Then equation (4) is replaced by

$$\nu = (2+2k)/(4+D'). \tag{5}$$

If an argument analogous to equation (5) is used for a SAW on a lattice, we get a result similar to that of Rammal *et al* (1984, see also Sahimi 1984) which seems to contradict Monte Carlo data (Kremer 1981, Lyklema and Kremer 1984). The analogue of equation (4) for a SAW, on the other hand, is Kremer's empirical rule $3/(2+D')$, which agrees with his Monte Carlo data.

In the limiting case where the plaquettes are distributed in an euclidian space of dimensionality d (instead of being restricted on IIC) $D' = d$ and $k = \frac{1}{2}$; substituting these values in equation (4) or (5) we obtain,

$$\nu = 3/(d+4) \tag{6}$$

which is the result of Maritan and Stella (1984). The fractal dimensionality D_f of the SAS embedded in a fractal can then be obtained from $D_f = 1/\nu$. To find the upper dimensionality above which the excluded volume interactions become negligible, we require $F_{rep} \rightarrow 0$ as $N \rightarrow \infty$, i.e. $R^{D'}/N^2$ should diverge. That seems impossible for the incipient infinite fractal at the percolation threshold where D' cannot exceed 4. Thus the above approximate analysis suggests that even for arbitrarily high dimensions d the SAS embedded on the incipient infinite network will always feel their repulsion, in contrast to the SAS embedded on regular lattices with dimensionality d above 8 (Maritan and Stella 1984).

(ii) *Fractal random surfaces in euclidean space.* In the above analysis we considered the random distribution of two-dimensional plaquettes restricted to lie on a fractal. Now we consider the surface A_f of a finite fractal object of dimensionality D embedded in a regular lattice of dimensionality d. This surface of the fractal is assumed to vary as

$$A_f \propto R^{D-1}. \tag{7}$$

N such finite objects cluster together with the restriction that their surfaces do not overlap (R being the radius of the fractal). The random fractal surfaces so assumed are gaussian distributed in a d-dimensional euclidian space to explore its conformations. In a sense this is opposite to our previous case of euclidean surfaces in a fractal space. As before, the linear extension R of the self-avoiding fractal random surfaces is determined by minimising the competing effects of attractive (elastic) free energy and repulsive (excluded volume) free energy. The elastic free energy is given by

$$F_{el} \propto R^{2(D-1)}/N \tag{8}$$

where we have extrapolated the arguments of Maritan and Stella (1984) replacing their planar plaquette by a fractal surface. The repulsive free energy is estimated as usual

$$F_{\text{rep}} \propto N^2/R^d. \tag{9}$$

Minimising $F = F_{\text{rep}} + F_{\text{el}}$ with respect to R we obtain

$$R \propto N^{\nu} \tag{10}$$

$$\nu \propto 3/(2D + d - 2) \tag{11}$$

which reduces to the results of Maritan and Stella (1984) for the self-avoiding random surfaces for plaquette for which $D = d = 3$. As another example, a percolation cluster ($D = 2.5$) in a cubic lattice ($d = 3$) gives $\nu = \frac{1}{2}$. The repulsive energy is negligible for d above some upper dimensionality,

$$d_c = 4D - 4 \tag{12}$$

which reproduces the result $d_c = 8$ for $D = d = 3$ in the limiting case of Maritan and Stella (1984).

(iii) *Fractal random surfaces embedded in the fractal.* Here we look at the above mentioned fractal surfaces of model (ii) embedded in the fractal of model (i) of dimensionality D'. Under the approximation outlined in preceeding sections the total free energy is the sum of the elastic energy of model (ii) and the repulsive energy of model (i):

$$\text{Constant}(1) R^{2(D-1)}/N + \text{Constant}(2) N^2/R^{D'}. \tag{13}$$

Minimisation with respect to R gives

$$R \propto N^{\nu}, \qquad \nu = 3/(2D + D' - 2). \tag{14}$$

As an example, a two-dimensional diffusion-limited aggregate (Witten and Sander 1983) embedded into a two-dimensional incipient percolating network has $\nu = 0.9$. As in the alternative, equation (5), in model (i) one may also replace 3 in equation (14) by $2 + 2k$ for this model. Again, there seems to be no upper dimension for the fractal surfaces embedded in the incipient percolating cluster.

In summary, we have presented several possible definitions of self-avoiding surfaces for the fractal objects and have given some speculative approximate results for their asymptotic linear extent R.

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